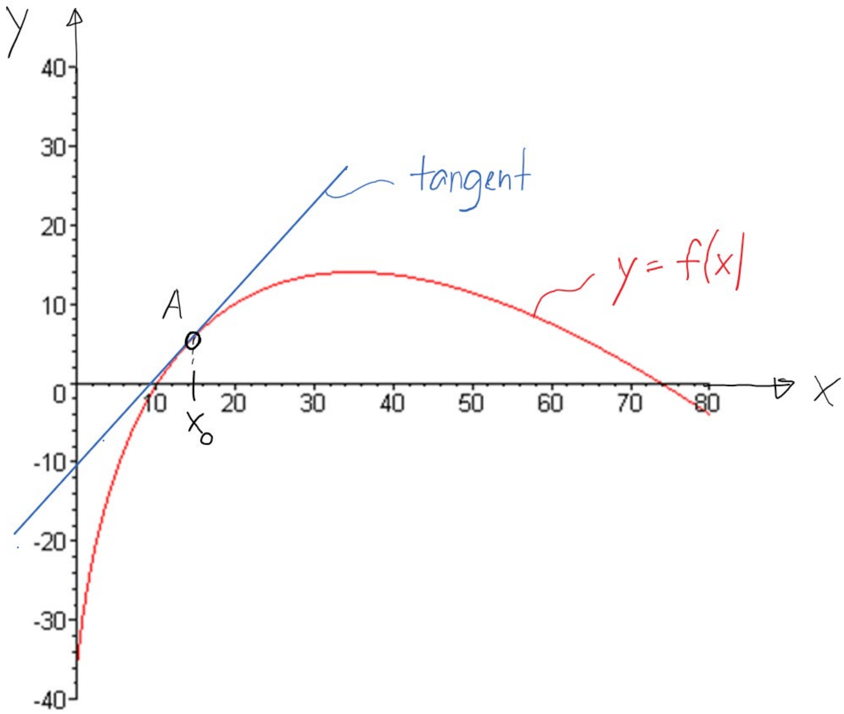
Derivative

Function f

f: $D \rightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}$
$x \mapsto y = f(x)$

Ex.: $f(x) = 24\sqrt{x+1} - 2x - 60$



What do we want to know?

Slope of the tangent to the graph of the function f at a certain point $A(x_0 \mid f(x_0))$.

Why do we want to know the slope?

- **rate of change** of values y with respect to x
- **increasing** (slope > 0), **decreasing** (slope < 0)
- local **maximum/minimum** (slope = 0)
- **concavity** (concave up if slope increases, concave down if slope decreases), points of inflection

Applications in economics

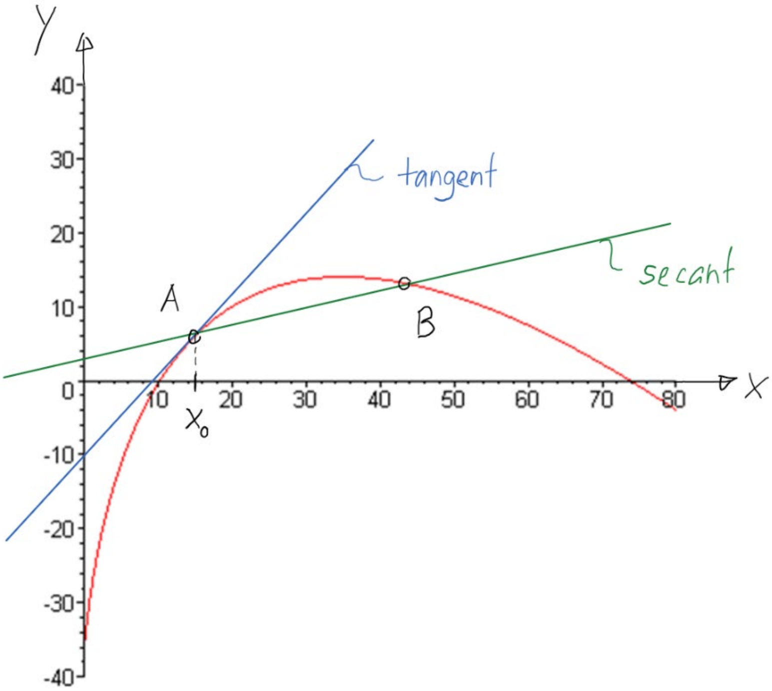
- tendency of costs/revenue/profit
- maximum/minimum of costs/revenue/profit
- **marginal costs/revenue/profit**: change of costs/revenue/profit if x (number of items or quantity of service) increases by one unit

Definition

The slope of the tangent to the graph of f through the point $A(x_0 \mid f(x_0))$ is called the **derivative** (or **rate of change**) **of f at the position x_0**, denoted $f'(x_0)$ ("f prime of x_0").

How can we determine the slope?

The slope of the **secant** through the points $A(x_0 \mid f(x_0))$ and $B(x_0+\Delta x \mid f(x_0+\Delta x))$ tends towards the slope of the **tangent** through $A(x_0 \mid f(x_0))$ as Δx tends towards 0.



Ex.: f: $\mathbb{R} \to \mathbb{R}$

$x \mapsto y = f(x) = x^2$

$f'(x_0) = 2x_0$ (without proof)

Ex.: f: $D \to \mathbb{R}$

$x \mapsto y = f(x) = 24\sqrt{x+1} - 2x - 60$

$f'(x_0) = \frac{12}{\sqrt{x_0+1}} - 2$ (without proof)

Definition

Suppose that the derivative (rate of change) $f'(x_0)$ exists for all $x_0 \in D_1$, where $D_1 \subseteq D$.

The function f'

$$f': D_1 \to \mathbb{R}$$
$$x \mapsto y = f'(x)$$

is called the **derivative** (or **derived function**) **of f**.

Ex.: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = x^2$ $\qquad$ $f': \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f'(x) = 2x$

Ex.: $f: D \to \mathbb{R}$, $x \mapsto y = f(x) = 24\sqrt{x+1} - 2x - 60$ $\qquad$ $f': D_1 \to \mathbb{R}$, $x \mapsto y = f'(x) = \frac{12}{\sqrt{x+1}} - 2$