Formulary

1. Algebra

Binomial formulae

$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ $(a + b)(a - b) = a^2 - b^2$

Powers and roots

$a^n \cdot a^m = a^{n+m}$ $\frac{a^n}{a^m} = a^{n-m}$

$(a^n)^m = a^{n \cdot m} = a^{m \cdot n} = (a^m)^n$

$a^{-n} = \frac{1}{a^n}$

$a^{1/n} = \sqrt[n]{a}$ $a^{m/n} = \sqrt[n]{a^m}$

$a^n \cdot b^n = (a \cdot b)^n$ $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

2. Logarithms

$a^x = y \iff x = \log_a(y)$

$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$

$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

$\log_a(u^v) = v \cdot \log_a(u)$

3. Functions and equations

Linear function

$y = f(x) = ax + b$ slope $a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Quadratic function (graph = parabola)

$y = f(x) = ax^2 + bx + c$ general form

$y = f(x) = a(x - u)^2 + v$ vertex form, vertex $V(u|v)$

Quadratic equation

$ax^2 + bx + c = 0$ general form

$\Rightarrow \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ quadratic formula

Exponential function

$y = f(x) = c \cdot a^x$ a = growth factor

4. Financial mathematics

r	interest rate (with regard to corresponding compounding period)
n	number of compounding periods
q	interest/growth factor, $q = 1 + r$
C_0	initial capital
C_n	capital after n compounding periods
Ordinary annuity	payments at the **end** of each compounding period
Annuity due	payments at the **beginning** of each compounding period
p	payment at the end/beginning of each compounding period
A_n	annuity value after n compounding periods (if initial value = 0)
A_0	initial annuity value (if value after n compounding periods = 0)

Interest

Simple interest

$$C_n = C_0\,(1 + nr) \quad\Rightarrow\quad C_0 = \frac{C_n}{1+nr} \qquad r = \frac{\frac{C_n}{C_0} - 1}{n} \qquad n = \frac{\frac{C_n}{C_0} - 1}{r}$$

Compound interest

$$C_n = C_0\, q^n \quad\Rightarrow\quad C_0 = \frac{C_n}{q^n} \qquad q = \sqrt[n]{\frac{C_n}{C_0}} \qquad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(q)}$$

$$C_n = C_0\,(1 + r)^n \quad\Rightarrow\quad C_0 = \frac{C_n}{(1+r)^n} \qquad r = \sqrt[n]{\frac{C_n}{C_0}} - 1 \qquad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(1+r)}$$

Annuity

Ordinary annuity

$$A_n = p\,\frac{q^n - 1}{q - 1} \quad\Rightarrow\quad p = \frac{A_n(q-1)}{q^n - 1} \qquad n = \frac{\log_a\left(\frac{A_n(q-1)}{p} + 1\right)}{\log_a(q)}$$

$$A_0 = p\,\frac{q^n - 1}{q^n(q - 1)} \quad\Rightarrow\quad p = \frac{A_0\,q^n(q-1)}{q^n - 1} \qquad n = \frac{\log_a\left(\frac{p}{p - A_0(q-1)}\right)}{\log_a(q)}$$

Annuity due

$$A_n = pq\,\frac{q^n - 1}{q - 1} \quad\Rightarrow\quad p = \frac{A_n(q-1)}{q(q^n - 1)} \qquad n = \frac{\log_a\left(\frac{A_n(q-1)}{pq} + 1\right)}{\log_a(q)}$$

$$A_0 = p\,\frac{q^n - 1}{q^{n-1}(q - 1)} \quad\Rightarrow\quad p = \frac{A_0\,q^{n-1}(q-1)}{q^n - 1} \qquad n = \frac{\log_a\left(\frac{pq}{pq - A_0(q-1)}\right)}{\log_a(q)}$$

5. Differential calculus

Derivative	$f'(x) = \lim\limits_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$		
Constant function	$f(x) = c \quad (c \in \mathbb{R})$	$\Rightarrow$	$f'(x) = 0$
Power function	$f(x) = x^n \quad (n \in \mathbb{R})$	$\Rightarrow$	$f'(x) = n \cdot x^{n-1}$
Exponential function	$f(x) = a^x \quad (a \in \mathbb{R}^+ \setminus \{1\})$	$\Rightarrow$	$f'(x) = a^x \cdot \ln(a)$
	$f(x) = e^x$	$\Rightarrow$	$f'(x) = e^x$
	$f(x) = e^{f_1(x)}$	$\Rightarrow$	$f'(x) = f_1'(x) \cdot e^{f_1(x)}$
Coefficient rule	$f(x) = c \cdot f_1(x) \quad (c \in \mathbb{R})$	$\Rightarrow$	$f'(x) = c \cdot f_1'(x)$
Sum rule	$f(x) = f_1(x) \pm f_2(x)$	$\Rightarrow$	$f'(x) = f_1'(x) \pm f_2'(x)$
Product rule	$f(x) = f_1(x) \cdot f_2(x)$	$\Rightarrow$	$f'(x) = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$

6. Integral calculus

Constant function	$\int c \, dx = cx + C$	$(c \in \mathbb{R}, C \in \mathbb{R})$
Power function	$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$	$(n \in \mathbb{R} \setminus \{-1\}, C \in \mathbb{R})$
	$\int x^{-1} \, dx = \ln(\lvert x \rvert) + C$	$(C \in \mathbb{R})$
Exponential function	$\int e^x \, dx = e^x + C$	$(C \in \mathbb{R})$
	$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$	$(k \in \mathbb{R} \setminus \{0\}, C \in \mathbb{R})$
Coefficient rule	$\int c \cdot f(x) \, dx = c \cdot \int f(x) \, dx$	$(c \in \mathbb{R})$
Sum rule	$\int (f_1(x) \pm f_2(x)) \, dx = \int f_1(x) \, dx \pm \int f_2(x) \, dx$	
Definite integral	$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$	