Exponentialfunktion

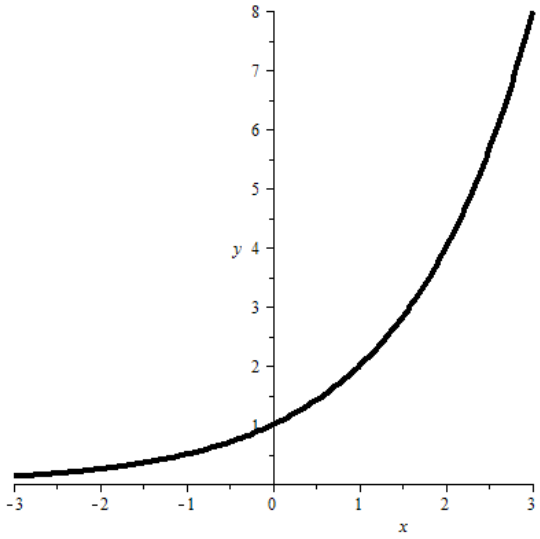
Definition

f:	$D \to \mathbb{R}$	$(D \subseteq \mathbb{R})$
	$x \mapsto y = f(x) = c \cdot a^x$	$(a \in \mathbb{R}^+ \setminus \{1\},\ c \in \mathbb{R} \setminus \{0\})$

$a > 1$: exponentielles **Wachstum**
$a < 1$: exponentieller **Zerfall**

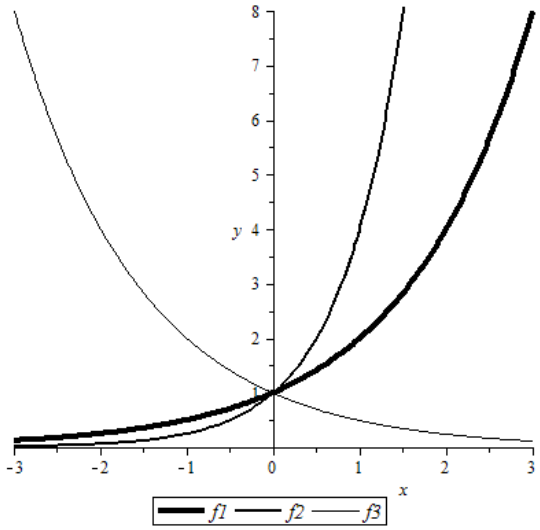
Graf

1. $y = f(x) = 2^x$ $(c = 1,\ a = 2)$



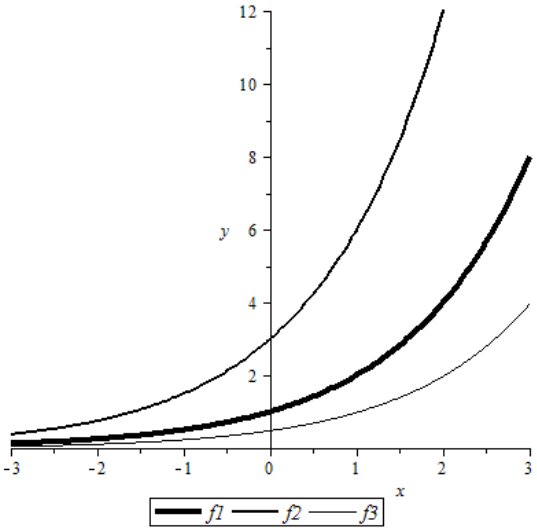
2. Parameter **a** (**a verändern**, c konstant halten)

$y = f_1(x) = 2^x$ $(c = 1,\ \mathbf{a = 2})$
$y = f_2(x) = 4^x$ $(c = 1,\ \mathbf{a = 4})$
$y = f_3(x) = \left(\frac{1}{2}\right)^x$ $\left(c = 1,\ \mathbf{a = \frac{1}{2}}\right)$

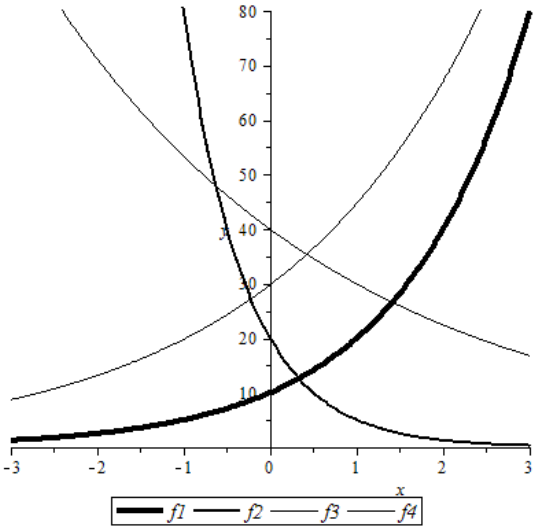


3. Parameter **c** (**c verändern**, a konstant halten)

$y = f_1(x) = 2^x$ (**c = 1**, a = 2)
$y = f_2(x) = 3 \cdot 2^x$ (**c = 3**, a = 2)
$y = f_3(x) = \frac{1}{2} \cdot 2^x$ $\left(\mathbf{c = \frac{1}{2}}, a = 2\right)$



4. $y = f_1(x) = 10 \cdot 2^x$ (c = 10, a = 2)
$y = f_2(x) = 20 \cdot 0.25^x$ (c = 20, a = 0.25)
$y = f_3(x) = 40 \cdot 0.75^x$ (c = 40, a = 0.75)
$y = f_4(x) = 30 \cdot 1.5^x$ (c = 30, a = 1.5)



Beispiele

1. Zinseszins (exponentielles **Wachstum**)

$K_n = K_0 \cdot q^n$

K_0 = Anfangskapital
K_n = Kapital nach n Zinsperioden
n = Anzahl Zinsperioden (häufig: 1 Zinsperiode = 1 Jahr)
q = Zins-/Wachstumsfaktor = 1 + i (i > 0, q > 1)
i = Zinssatz pro Zinsperiode

Bsp.: $K_0 := 1000$, $i := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow K_n = 1000 \cdot 1.02^n$

2. Preisindex (exponentieller **Zerfall**)

$P(t) = P_0 \cdot q^t$

P_0 = Anfangspreis / anfängliche Kaufkraft
P(t) = Preis / Kaufkraft zum Zeitpunkt t (häufig: t in Jahren)
q = Zerfallsfaktor = 1 + i (i < 0, q < 1)

Bsp.: $P_0 := 100$, $i := -3\% = -0.03 \Rightarrow q = 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$