

## Exercise 8

### Quadratic function and equations Quadratic function/equations, supply, demand, market equilibrium

#### Objectives

- be able to solve special quadratic equations without applying the quadratic formula.
- be able to solve a quadratic equation by applying the quadratic formula.
- be able to solve a quadratic equation containing a parameter.
- be able to determine the vertex form of a quadratic function out of the coordinates of the vertex and the coordinates of another point of the corresponding parabola.
- be able to determine the general form of a quadratic function out of the coordinates of three points of the corresponding parabola.
- be able to treat applied tasks in economics by means of quadratic equations or systems of quadratic equations.

#### Problems

1. Solve the quadratic equations below without using the quadratic formula.

State the solution set for each equation.

a)  $(x + 2)(x + 5) = 0$

b)  $(x - 8)(5x - 9) = 0$

c)  $x^2 - 3x = 0$

d)  $x^2 + 7x = 0$

e)  $4x^2 - 9 = 0$

f)  $100x^2 - 1 = 0$

g)  $(3x - 2)(4x + 1) = 0$

h)  $4x^2 + 5x = 0$

i)  $3x^2 = 27$

j)  $x^2 = x$

2. Solve the quadratic equations below. State the solution set for each equation.

a)  $(7 + x)(7 - x) = (3x + 2)^2 - (2x + 3)^2$

b)  $(x - 3)(2x - 7) = 1$

c)  $\frac{8}{x^2 - 4} + \frac{2}{2 - x} = 3x - 1$

d)  $\frac{x - 4}{x - 5} = \frac{30 - x^2}{x^2 - 5x}$

e)  $\frac{x^2 - x - 2}{2 - x} = 1$

f)  $\frac{x^2 - 4}{x^2 - 4} = 1$

3. Determine the value(s) of the parameter b such that the quadratic equation has exactly one solution.  
State this solution:

a)  $2x^2 = 3x - b$

b)  $x^2 + bx + b = -3$

4. Solve the following equations for x. Take into account that the parameter b can have any real value.

a)  $x^2 + x + b = 0$

b)  $-bx = 1 + 4x^2$

5. A parabola has the vertex V and contains the point P.

Determine the formula of the corresponding quadratic function both in the vertex and in the general form.

a)  $V(2|4) \quad P(-1|7)$

b)  $V(1|-8) \quad P(2|-7)$

6. A parabola contains the three points P, Q, and R.

Determine the formula of the corresponding quadratic function in the general form.

- a) P(-4|8)      Q(0|0)      R(10|15)  
b) P(1|-1)      Q(2|4)      R(4|8)

7. Find the equilibrium quantity and equilibrium price of a commodity for the given supply and demand functions  $f_s$  and  $f_d$ :

- a) supply       $p = f_s(q) = \frac{1}{4}q^2 + 10$   
demand       $p = f_d(q) = 86 - 6q - 3q^2$   
b) supply       $p = f_s(q) = q^2 + 8q + 16$   
demand       $p = f_d(q) = -3q^2 + 6q + 436$

8. The total costs and the total revenues for a company are given by

$$C(x) = 2000 + 40x + x^2$$
$$R(x) = 130x$$

Find the break-even points.

9. The costs  $C(x)$  for producing  $x$  items and the revenue  $R(x)$  for selling  $x$  items are given below.  
How many items are to be produced and sold in order to achieve a profit of 200 CHF?

$$C(x) = (x^2 + 100x + 80) \text{ CHF}$$
$$R(x) = (160x - 2x^2) \text{ CHF}$$

## Answers

- |    |    |                     |    |                       |
|----|----|---------------------|----|-----------------------|
| 1. | a) | $S = \{-5, -2\}$    | b) | $S = \{9/5, 8\}$      |
|    | c) | $S = \{0, 3\}$      | d) | $S = \{-7, 0\}$       |
|    | e) | $S = \{-3/2, 3/2\}$ | f) | $S = \{-1/10, 1/10\}$ |
|    | g) | $S = \{-1/4, 2/3\}$ | h) | $S = \{-5/4, 0\}$     |
|    | i) | $S = \{-3, 3\}$     | j) | $S = \{0, 1\}$        |

2. a)  $S = \{-3, 3\}$  b)  $S = \{5/2, 4\}$   
c)  $S = \{-5/3, 0\}$  d)  $S = \{-3\}$   
e)  $S = \{-1\}$  f)  $S = \{ \}$

- $$3. \quad \begin{array}{ll} a) & b = \frac{9}{8} \\ b) & b_1 = -2 \\ & b_2 = 6 \end{array} \quad \begin{array}{l} x = \frac{3}{4} \\ x = 1 \\ x = - \end{array}$$

- |    |    |                   |  |             |
|----|----|-------------------|--|-------------|
| 4. | a) | $b < \frac{1}{4}$ | $x_{1,2} = \frac{-1 \pm \sqrt{1 - 4b}}{2}$   | 2 solutions |
|    |    | $b = \frac{1}{4}$ | $x = -\frac{1}{2}$                           | 1 solution  |
|    |    | $b > \frac{1}{4}$ | $S = \{ \}$                                  | no solution |
|    | b) | $ b  > 4$         | $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 16}}{8}$ | 2 solutions |
|    |    | $b = \pm 4$       | $x = -\frac{b}{8}$                           | 1 solution  |
|    |    | $ b  < 4$         | $S = \{ \}$                                  | no solution |

5. a)  $y = f(x) = \frac{1}{3}(x - 2)^2 + 4 = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{16}{3}$

6. a)  $y = f(x) = \frac{1}{4}x^2 - x$   
b)  $y = f(x) = -x^2 + 8x - 8$

7.      a) at market equilibrium:  $q = 4, p = 14$   
           b) at market equilibrium:  $q = 10, p = 196$

$$8. \quad x_1 = 40, x_2 = 50$$

$$9. \quad \text{profit } P(x) = R(x) - C(x) = -3x^2 + 60x - 80 \stackrel{!}{=} 200$$

$$S = \{7.41..., 12.58...\}$$

7 or 13 items