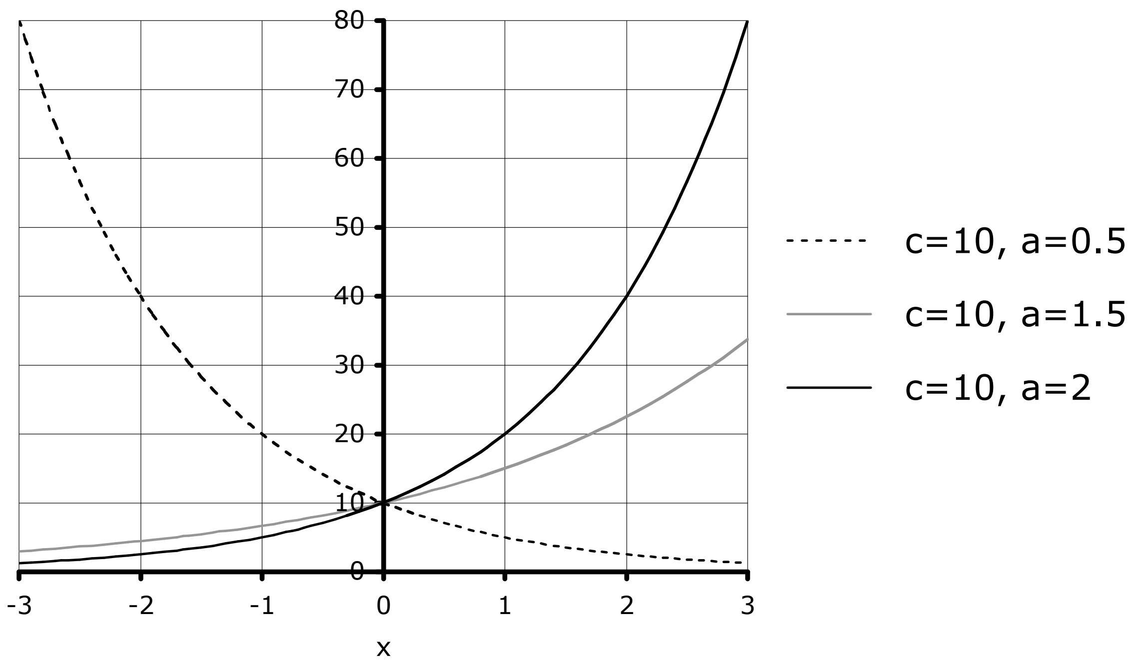
Exponential function

Definition

f: $\mathbb{R} \to \mathbb{R}$
$x \to y = f(x) = c \cdot a^x \quad (a \in \mathbb{R}^+ \setminus \{1\},\ c \in \mathbb{R} \setminus \{0\})$

a>1: exponential **growth**
a<1: exponential **decay**

Graph



Examples

1. Compound interest (exponential **growth**)

$C_n = C_0 \cdot q^n$

C_0 = initial capital
C_n = capital after n compounding periods
n = number of compounding periods (typically: 1 compounding period = 1 year)
q = growth factor = 1 + r (q>1)
r = interest rate per compounding period

Ex.: $C_0 := 1000$, $r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$

2. Consumer price index (exponential **decay**)

$P(t) = P_0 \cdot q^t$

P_0 = initial purchasing power
P(t) = purchasing power at time t (typically: t in years)
q = decay factor (q<1)

Ex.: $P_0 := 100$, $q := 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$