

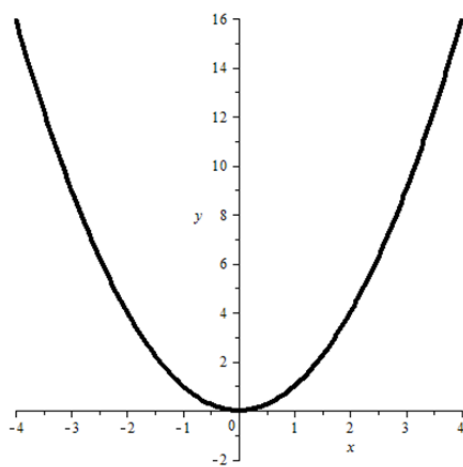
Quadratic function

Definition

$f: D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \rightarrow y = f(x) = ax^2 + bx + c$	$(a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}, c \in \mathbb{R})$
general form	
$y = f(x) = a(x - u)^2 + v$	$(a \in \mathbb{R} \setminus \{0\}, u \in \mathbb{R}, v \in \mathbb{R})$
vertex form	

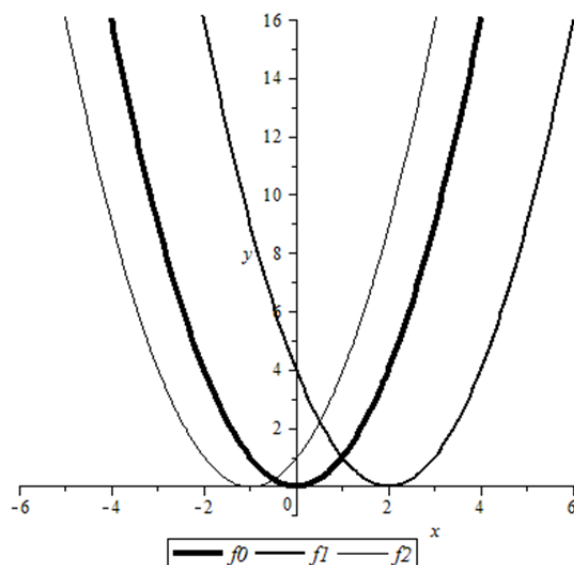
Graph

1. $y = f(x) = x^2$ $(a = 1, u = 0, v = 0)$



2. Parameter u $(a = 1, v = 0)$

$$\begin{aligned} y = f_0(x) &= x^2 & (u = 0) \\ y = f_1(x) &= (x - 2)^2 & (u = 2) \\ y = f_2(x) &= (x + 1)^2 & (u = -1) \end{aligned}$$

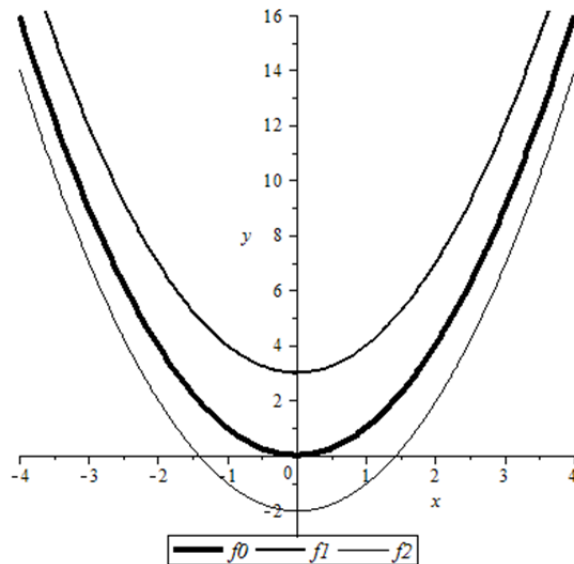


3. Parameter v ($a = 1, u = 0$)

$$y = f_0(x) = x^2 \quad (v = 0)$$

$$y = f_1(x) = x^2 + 3 \quad (v = 3)$$

$$y = f_2(x) = x^2 - 2 \quad (v = -2)$$

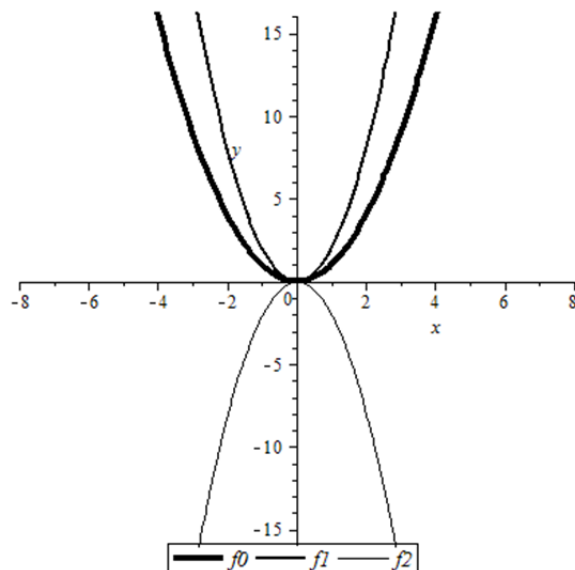


4. Parameter a ($u = 0, v = 0$)

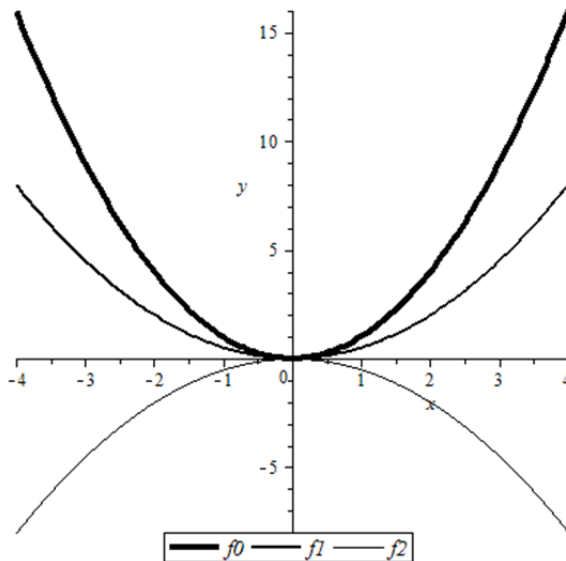
$$y = f_0(x) = x^2 \quad (a = 1)$$

$$y = f_1(x) = 2x^2 \quad (a = 2)$$

$$y = f_2(x) = -2x^2 \quad (a = -2)$$



$$\begin{aligned} y = f_0(x) &= x^2 & (a = 1) \\ y = f_1(x) &= \frac{1}{2}x^2 & \left(a = \frac{1}{2}\right) \\ y = f_2(x) &= -\frac{1}{2}x^2 & \left(a = -\frac{1}{2}\right) \end{aligned}$$

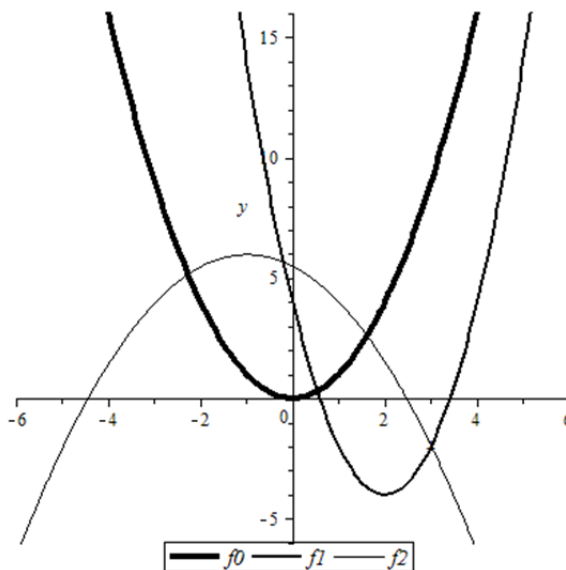


5. The **graph** of a quadratic function is a **parabola**.

The parameter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** V of the parabola: $V(u|v)$

$$\begin{aligned} y = f_0(x) &= x^2 & (a = 1, u = 0, v = 0) & V(0|0) \\ y = f_1(x) &= 2(x - 2)^2 - 4 & (a = 2, u = 2, v = -4) & V(2|-4) \\ y = f_2(x) &= -\frac{1}{2}(x + 1)^2 + 6 & \left(a = -\frac{1}{2}, u = -1, v = 6\right) & V(-1|6) \end{aligned}$$

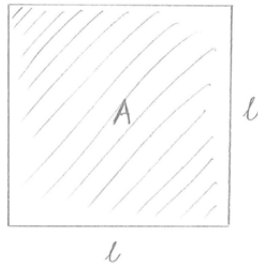


Examples

1. Nature/Physics: Trajectories of water in a fountain



2. Geometry: Square

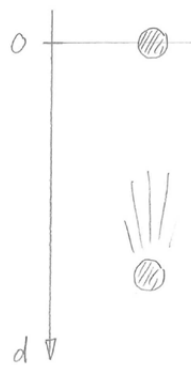


Area A for side length l : $A = l^2$

$f: \mathbb{R}^+ \rightarrow \mathbb{R}$

$l \rightarrow A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t : $d = \frac{1}{2}gt^2$ (g = gravity field strength)

$f: \mathbb{R} \rightarrow \mathbb{R}$

$t \rightarrow d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand