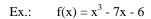
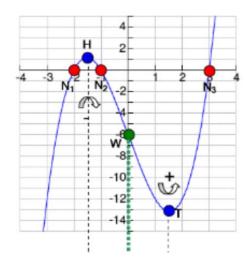
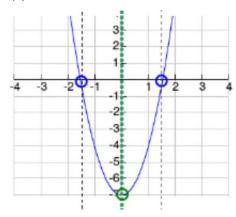
Increasing/decreasing, concavity

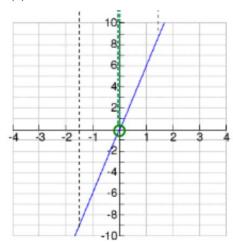








$$f''(x) = 6x$$



Increasing/decreasing

The function f is **increasing** at $x = x_0$, if the **first derivative** is **positive**, i.e. $f'(x_0) > 0$.

The function f is **decreasing** at $x = x_0$, if the **first derivative** is **negative**, i.e. $f'(x_0) < 0$.

Concavity

The graph of the function f is **concave up** at $x = x_0$, if the **second derivative** is **positive**, i.e. $f''(x_0) > 0$.

The graph of the function f is **concave down** at $x = x_0$, if the **second derivative** is **negative**, i.e. $f''(x_0) < 0$.

Relative maxima/minima

The function f has a **relative maximum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave down at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) < 0$.

The function f has a **relative minimum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave up at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) > 0$.

Absolute maximum/minimum

The **absolute maximum/minimum** of a continuous function f is either a relative maximum/minimum or the value of f at one of the endpoints of the domain.

Points of inflection

The function f has a **point of inflection** at $x = x_0$, if the graph of f changes its concavity from concave up to concave down (or vice versa) at $x = x_0$, i.e. if $f''(x_0) = 0$ and $f'''(x_0) \neq 0$.

Ex.:
$$f(x) = x^3 - 7x - 6$$
 (see page 1) $\Rightarrow f'(x) = 3x^2 - 7$
 $\Rightarrow f''(x) = 6x$
 $\Rightarrow f'''(x) = 6$

Relative maxima/minima

$$f'(x) = 0 \text{ at } x_1 = \sqrt{\frac{7}{3}} = 1.52... \text{ and } x_2 = -\sqrt{\frac{7}{3}} = -1.52...$$

$$f''(x_1) = 6 \cdot \sqrt{\frac{7}{3}} = 9.16... > 0 \qquad \Rightarrow \text{ relative minimum at } x_1 = \sqrt{\frac{7}{3}}$$

$$f''(x_2) = -6 \cdot \sqrt{\frac{7}{3}} = -9.16... < 0 \qquad \Rightarrow \text{ relative maximum at } x_2 = -\sqrt{\frac{7}{3}}$$

Absolute maximum/minimum

Ex.:
$$D = [0,4]$$
 \Rightarrow absolute maximum at $x = 4$ (endpoint of domain) \Rightarrow absolute minimum at $x = x_1 = \sqrt{\frac{7}{3}}$ (relative minimum) \Rightarrow absolute maximum at $x = x_2 = -\sqrt{\frac{7}{3}}$ (relative maximum) \Rightarrow absolute minimum at $x = -4$ (endpoint of domain)

Points of inflection

$$f''(x) = 0$$
 at $x_3 = 0$
 $f'''(x_3) = 6 \neq 0$ \Rightarrow point of inflection at $x_3 = 0$

Financial mathematics

Marginal cost/revenue/profit function = first derivative of the cost/revenue/profit function

 $C(x) = 120x + x^2$ Cost function C'(x) = 120 + 2x⇒ Marginal cost function Revenue function

 $R(x) = 168x - 0.2x^2$ ⇒ Marginal revenue function R'(x) = 168 - 0.4x

Profit function $P(x) = R(x) - C(x) = 48x - 1.2x^{2}$

⇒ Marginal profit function P'(x) = 48 - 2.4x

Average cost/revenue/profit function

 $\overline{C}(x) := \frac{C(x)}{x}$ where C(x) = cost function Average cost function

 $C(x) = 3x^2 + 4x + 2$ Ex.: Cost function \Rightarrow Average cost function $\overline{C}(x) = 3x + 4 + \frac{2}{x}$

 $\overline{R}(x) := \frac{R(x)}{x}$ where R(x) = revenue function Average revenue function

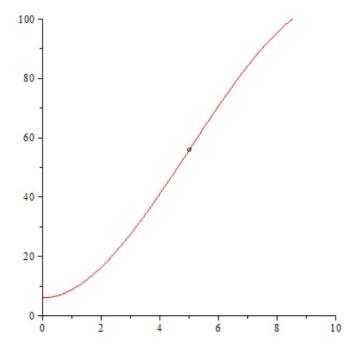
 $\bar{P}(x) := \frac{P(x)}{r}$ Average profit function where P(x) = profit function

Point of diminishing returns

Point of diminishing returns = point of inflection on the graph

Profit function Ex.:

$$P(x) = -0.2x^3 + 3x^2 + 6$$



Point of diminishing returns: (5|56)