Exponential function

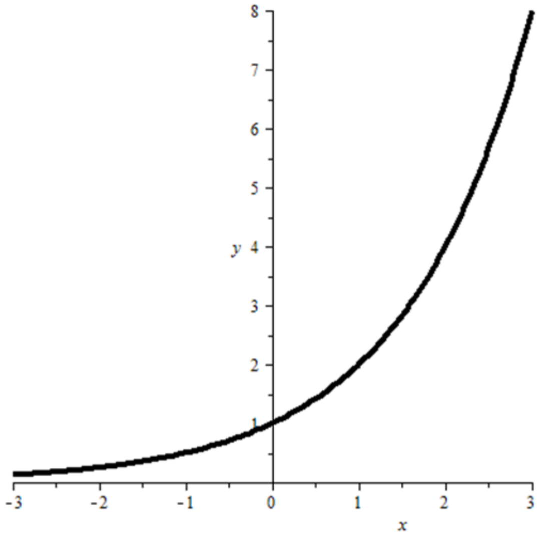
Definition

f: D $\rightarrow$ $\mathbb{R}$ $(D \subseteq \mathbb{R})$
x $\rightarrow$ $y = f(x) = c \cdot a^x$ $(a \in \mathbb{R}^+ \backslash \{1\},\ c \in \mathbb{R} \backslash \{0\})$

a > 1: exponential **growth**
a < 1: exponential **decay**

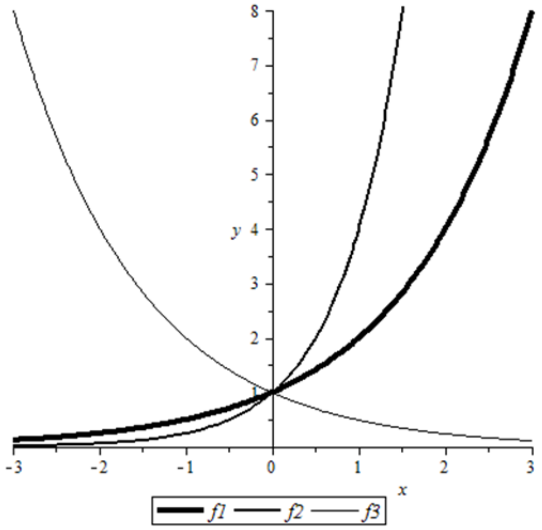
Graph

1. $y = f(x) = 2^x$ (c = 1, a = 2)



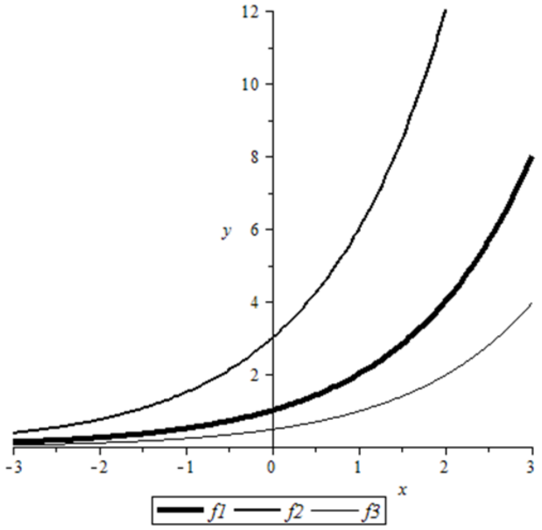
2. Parameter a

$y = f_1(x) = 2^x$ (c = 1, a = 2)
$y = f_2(x) = 4^x$ (c = 1, a = 4)
$y = f_3(x) = \left(\frac{1}{2}\right)^x$ $\left(c = 1, a = \frac{1}{2}\right)$

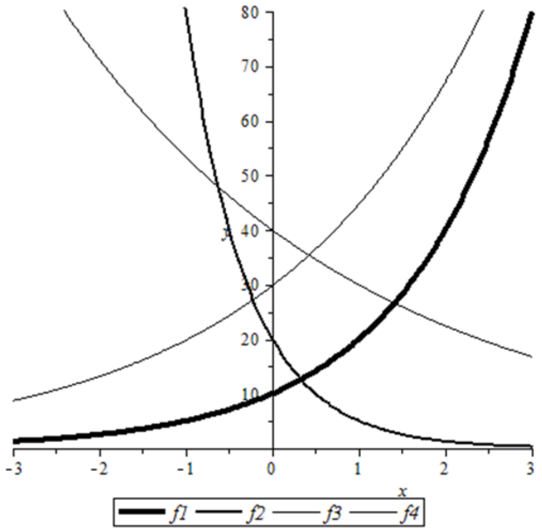


3. Parameter c

$y = f_1(x) = 2^x$ (a = 1, c = 1)
$y = f_2(x) = 3 \cdot 2^x$ (a = 1, c = 3)
$y = f_3(x) = \frac{1}{2} \cdot 2^x$ $\left(a = 1, c = \frac{1}{2}\right)$



4. $y = f_1(x) = 10 \cdot 2^x$ (c = 10, a = 2)
$y = f_2(x) = 20 \cdot 0.25^x$ (c = 20, a = 0.25)
$y = f_3(x) = 40 \cdot 0.75^x$ (c = 40, a = 0.75)
$y = f_4(x) = 30 \cdot 1.5^x$ (c = 30, a = 1.5)



Examples

1. Compound interest (exponential **growth**)

$C_n = C_0 \cdot q^n$

C_0 = initial capital
C_n = capital after n compounding periods
n = number of compounding periods (typically: 1 compounding period = 1 year)
q = growth factor = 1 + r (q > 1)
r = interest rate per compounding period

Ex.: $C_0 := 1000$, $r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$

2. Consumer price index (exponential **decay**)

$P(t) = P_0 \cdot q^t$

P_0 = initial purchasing power
P(t) = purchasing power at time t (typically: t in years)
q = decay factor (q < 1)

Ex.: $P_0 := 100$, $q := 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$