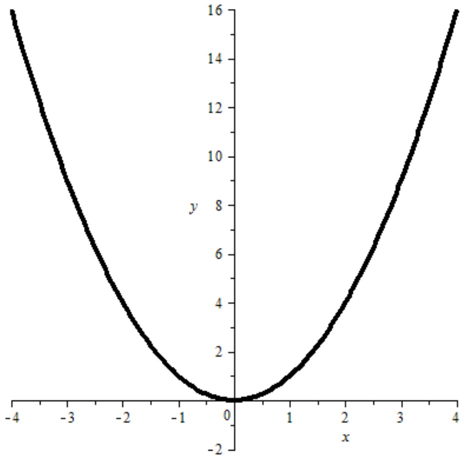
Quadratic function

Definition

f: D $\to \mathbb{R}$	($D \subseteq \mathbb{R}$)
x $\to$ y = f(x) = $ax^2 + bx + c$ general form	($a \in \mathbb{R}\backslash\{0\}$, $b \in \mathbb{R}$, $c \in \mathbb{R}$)
y = f(x) = $a(x - u)^2 + v$ vertex form	($a \in \mathbb{R}\backslash\{0\}$, $u \in \mathbb{R}$, $v \in \mathbb{R}$)

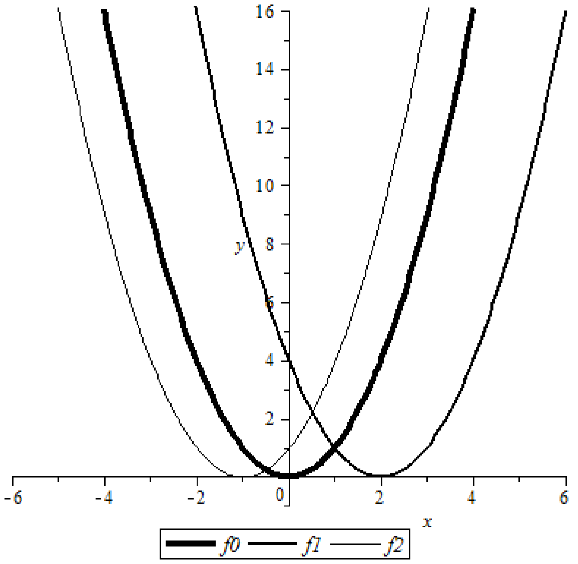
Graph

1. $y = f(x) = x^2$ (a = 1, u = 0, v = 0)



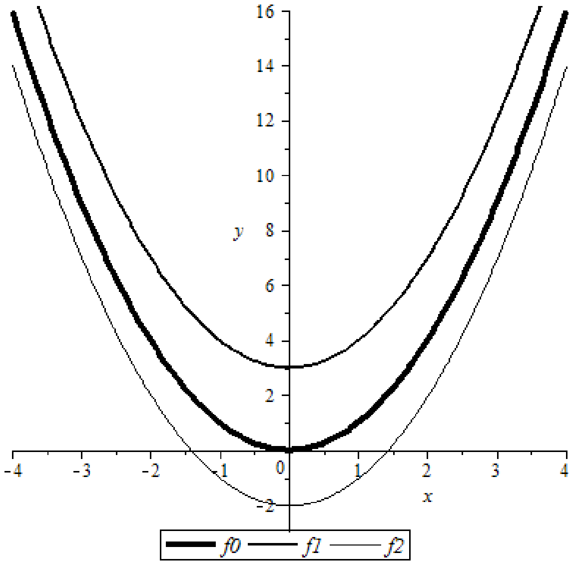
2. Parameter u (a = 1, v = 0)

$y = f_0(x) = x^2$ (u = 0)
$y = f_1(x) = (x - 2)^2$ (u = 2)
$y = f_2(x) = (x + 1)^2$ (u = -1)



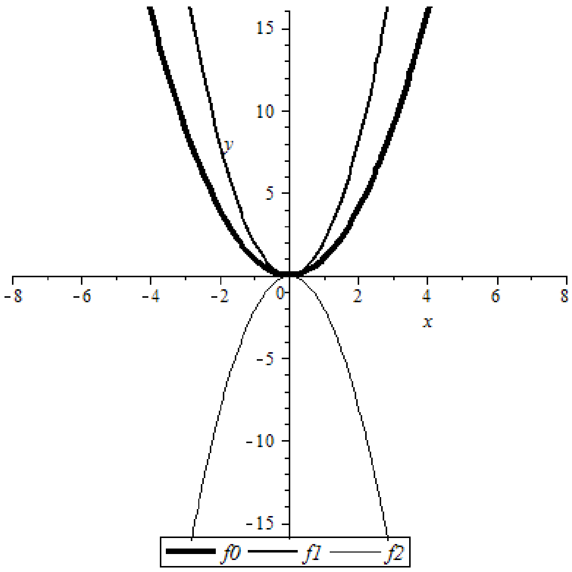
3. Parameter v (a = 1, u = 0)

$y = f_0(x) = x^2$ (v = 0)
$y = f_1(x) = x^2 + 3$ (v = 3)
$y = f_2(x) = x^2 – 2$ (v = -2)

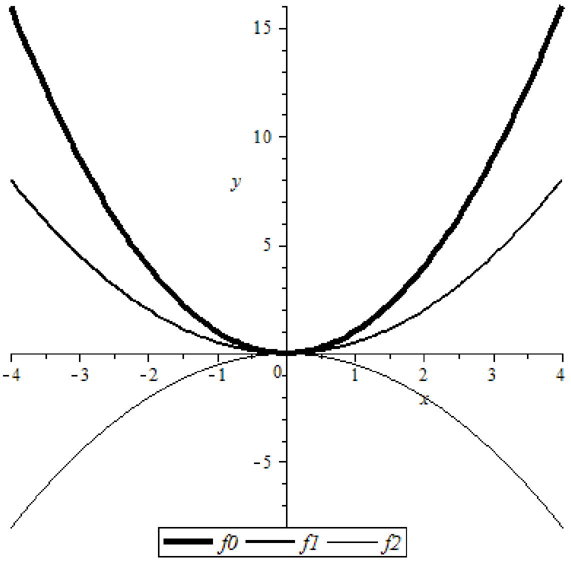


4. Parameter a (u = 0, v = 0)

$y = f_0(x) = x^2$ (a = 1)
$y = f_1(x) = 2x^2$ (a = 2)
$y = f_2(x) = -2x^2$ (a = -2)



$y = f_0(x) = x^2$ $(a = 1)$
$y = f_1(x) = \frac{1}{2}x^2$ $\left(a = \frac{1}{2}\right)$
$y = f_2(x) = -\frac{1}{2}x^2$ $\left(a = -\frac{1}{2}\right)$

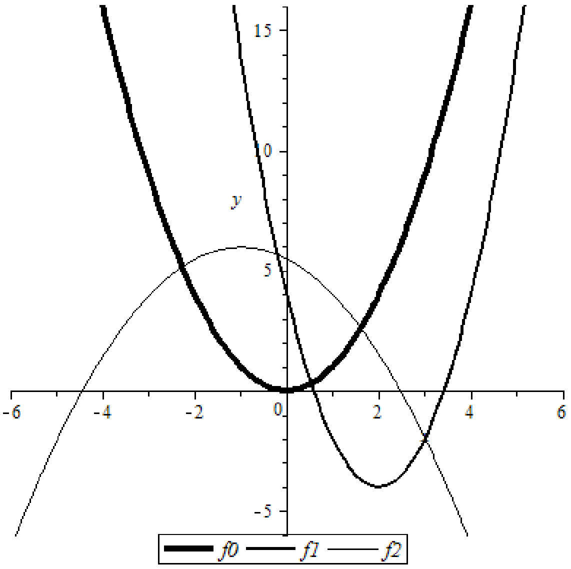


5. The **graph** of a quadratic function is a **parabola**.

The paramter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** V of the parabola: V(u|v)

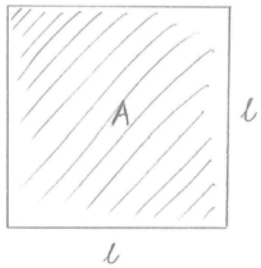
$y = f_0(x) = x^2$ $(a = 1, u = 0, v = 0)$ V(0|0)
$y = f_1(x) = 2(x-2)^2 - 4$ $(a = 2, u = 2, v = -4)$ V(2|-4)
$y = f_2(x) = -\frac{1}{2}(x+1)^2 + 6$ $\left(a = -\frac{1}{2}, u = -1, v = 6\right)$ V(-1|6)



Examples

1. Nature/Physics: Trajectories of water in a fountain

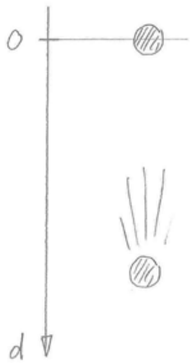
2. Geometry: Square



Area A for side length l: $A = l^2$

f: $\mathbb{R}^+ \to \mathbb{R}$
$l \to A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t: $d = \frac{1}{2}gt^2$ (g = gravity field strength)

f: $\mathbb{R} \to \mathbb{R}$
$t \to d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand