

Exercises 11 Exponential function and equations Compound interest, nominal/effective annual interest rate

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded more than once per year.
- know and understand the terms "nominal annual interest rate" and "effective annual interest rate".
- be able to treat specific compound interest tasks.

Problems

- 11.1 An initial capital $C_0 = 1000$ CHF is invested at a nominal annual interest rate $r = 10\%$, compounded ...
- a) ... quarterly.
 - i) Determine the capitals C_1 , C_2 , and C_3 , after one, two, and three years respectively.
 - ii) Determine the effective annual interest rate r^* .
 - b) ... monthly.
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 - ii) Determine the effective annual interest rate r^* .
- 11.2 Determine the effective annual interest rate for a nominal annual interest rate of 6% , compounded ...
- a) ... annually.
 - b) ... semiannually.
 - c) ... quarterly.
 - d) ... monthly.
 - e) ... daily (1 year = 360 days).
- 11.3 What is the future value if \$3200 is invested for 5 years at 8% compounded quarterly?
- 11.4 Find the interest that will be earned if \$10'000 is invested for 3 years at 9% compounded monthly.
- 11.5 What amount of money do parents need to deposit in an account earning 10% , compounded monthly, so that it will grow to \$40'000 for their son's college tuition in 18 years?
- 11.6 An initial capital of 1000 CHF amounts to 1500 CHF if it is invested for 10 years at an unknown annual interest rate, compounded quarterly.
- Determine the ...
- a) ... nominal annual interest rate.
 - b) ... effective annual interest rate.
- 11.7 How long (in months) would a capital have to be invested at 6% , compounded monthly, to double its value?

- 11.8 Ms P. wants to invest 100'000 CHF. Her bank makes two offers:
- A effective annual interest rate of 8.5%
 - B nominal annual interest rate of 8%, compounded monthly
- Which offer is better, offer A or offer B?
- 11.9 How long (in years) would 1000 CHF have to be invested at 2.5%, compounded daily, to earn 250 CHF interest?
- 11.10 At what nominal rate, compounded quarterly, would \$20'000 have to be invested to amount to \$26'425.82 in 7 years?
- 11.11 A couple needs \$15'000 as a down payment for a home. If they invest the \$10'000 they have at 8% compounded quarterly, how long will it take for the money to grow into \$15'000?
- 11.12 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.
- a) The nominal interest rate ...
- ... is generally higher than the effective interest rate.
 - ... is equal to the effective interest rate if interest is compounded annually.
 - ... is half as much as the effective interest rate if interest is compounded semiannually.
 - ... depends on the compounding period.
- b) In a compound interest scheme where interest is compounded m ($m > 1$) times per year ...
- ... the growth factor is m times as high as if interest is compounded only once per year.
 - ... the annual interest rate is m times lower than if interest is compounded only once per year.
 - ... the capital grows faster than if interest is compounded only once per year.
 - ... the capital grows more slowly than if interest is compounded only once per year.
- c) If an initial capital of 1000 CHF grows to 1100 CHF in one year and interest is compounded semiannually ...
- ... the effective interest rate is less than 10%.
 - ... the effective interest rate is greater than 10%.
 - ... the nominal interest rate is less than 10%.
 - ... the nominal interest rate is greater than 10%.

Answers

11.1 a) i) $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$
 $C_1 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 1} \text{ CHF} = 1103.81 \text{ CHF (rounded)}$
 $C_2 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 2} \text{ CHF} = 1218.40 \text{ CHF (rounded)}$
 $C_3 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 3} \text{ CHF} = 1344.89 \text{ CHF (rounded)}$
 ii) $r^* = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.1038 = 10.38\% \text{ (rounded)}$

b) i) $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$
 $C_1 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 1} \text{ CHF} = 1104.71 \text{ CHF (rounded)}$
 $C_2 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 2} \text{ CHF} = 1220.39 \text{ CHF (rounded)}$
 $C_3 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 3} \text{ CHF} = 1348.18 \text{ CHF (rounded)}$
 ii) $r^* = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\% \text{ (rounded)}$

11.2 $r^* = \left(1 + \frac{r}{m}\right)^m - 1$ $r = 6\% = 0.06$

a) $m = 1$ $r^* = 6\%$
 b) $m = 2$ $r^* = 6.09\%$
 c) $m = 4$ $r^* = 6.136\% \text{ (rounded)}$
 d) $m = 12$ $r^* = 6.168\% \text{ (rounded)}$
 e) $m = 360$ $r^* = 6.183\% \text{ (rounded)}$

11.3 $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$ where $C_0 = \$3200, r = 8\%, m = 4, n = 5$
 $\Rightarrow C_5 = \$4755.03 \text{ (rounded)}$

11.4 Interest = $C_n - C_0$
 $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$ where $C_0 = \$10'000, r = 9\%, m = 12, n = 3$
 $\Rightarrow C_n - C_0 = \$3086.45 \text{ (rounded)}$

11.5 $C_0 = \frac{C_n}{\left(1 + \frac{r}{m}\right)^{mn}}$ where $C_n = \$40'000, r = 10\%, m = 12, n = 18$
 $\Rightarrow C_0 = \$6661.46 \text{ (rounded)}$

11.6 a) $r = m \left(\sqrt[mn]{\frac{C_n}{C_0}} - 1 \right)$ where $C_0 = \$1000, C_n = \$1500, m = 4, n = 10$
 $\Rightarrow r = 4.08\% \text{ (rounded)}$
 b) $r^* = \left(1 + \frac{r}{m}\right)^m - 1$
 $\Rightarrow r^* = 4.14\% \text{ (rounded)}$

11.7 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where $\frac{C_n}{C_0} = 2$, $r = 6\%$, $m = 12$, $a := 10$ (any $a \in \mathbb{R}^+ \setminus \{1\}$ would be possible)
 $\Rightarrow n = 11.58\dots$
 $\Rightarrow mn = 138.98\dots \rightarrow 139 \text{ months} = 11 \text{ years } 7 \text{ months}$

11.8 A $r^*(A) = 8.5\%$
B $r^*(B) = \left(1 + \frac{r}{m}\right)^m - 1$ where $r = 8\%$, $m = 12$
 $\Rightarrow r^*(B) = 8.3\%$
 $\Rightarrow r^*(A) > r^*(B)$, i.e. offer A is better than offer B

11.9 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where $C_0 = 1000 \text{ CHF}$, $C_n = 1250 \text{ CHF}$, $r = 2.5\%$, $m = 360$, $a := 10$
 $\Rightarrow n = 8.92\dots \rightarrow 9 \text{ years}$

11.10 $r = m \left(\sqrt[mn]{\frac{C_n}{C_0}} - 1 \right)$ where $C_0 = \$20'000$, $C_n = \$26'425.82$, $m = 4$, $n = 7$
 $\Rightarrow r = 4\%$

11.11 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where $C_0 = \$10'000$, $C_n = \$15'000$, $r = 8\%$, $m = 4$, $a := 10$
 $\Rightarrow n = 5.11\dots$
 $\Rightarrow mn = 20.47\dots \rightarrow 21 \text{ quarters} = 5 \text{ years } 3 \text{ months}$

- 11.12 a) 2nd statement
b) 3rd statement
c) 3rd statement