Exercises 3 Function
Domain, codomain, range, graph

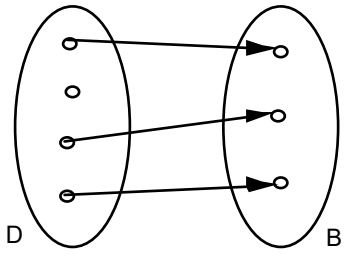
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

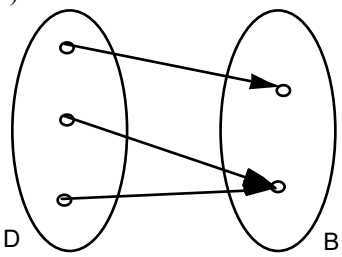
Problems

3.1 Which of the following relations are functions? Explain your answer.

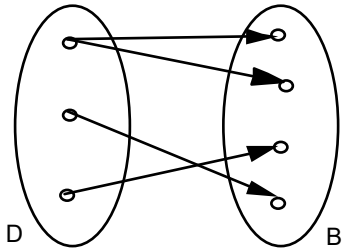
a)



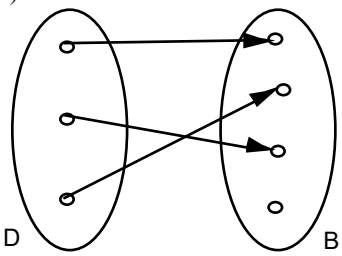
b)



c)



d)



e) D = set of all modules of the FHGR Tourism bachelor programme
B = set of all FHGR lecturers
f: $D \to B$, $m \mapsto l = f(m)$ = lecturer of m

f) D = {1989, 1990, ... , 1998, 1999}
B = set of all human beings aged between 20 and 30
f: $D \to B$, $y \mapsto p = f(y)$ = person who was born in the year y

g) D = set of all human beings aged between 20 and 30
B = {1989, 1990, ... , 1998, 1999}
f: $D \to B$, $p \mapsto y = f(p)$ = year of birth of person p

h) f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = x^2$

i) f: $\mathbb{R}^+ \to \mathbb{R}$, $x \mapsto y = f(x)$ = number whose square is x

Notice:
- $\mathbb{R}^+$ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.

j) f: $\mathbb{R} \to \mathbb{R}$, $t \mapsto b = f(t)$ = bank account balance at time t

3.2 (see next page)

3.2 Determine the range E of the functions below:

a) D = {January, February, March, ..., December}
B = {A, B, C, ..., Z}
f: $D \to B$, $m \mapsto l = f(m)$ = initial letter of m

b) D = set of all neighbouring countries of Switzerland
B = set of all European cities
c: $D \to B$, $x \mapsto y = c(x)$ = capital of neighbouring country x

c) function f in problem 3.1 g)

d) function f in problem 3.1 h)

3.3 a) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto f(x) = x^3 - x$

Determine the following values:

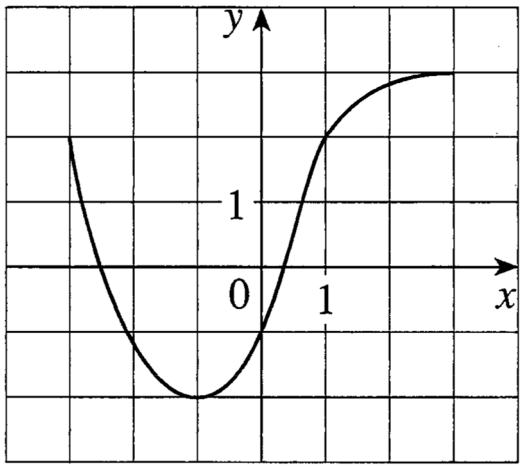
i)	f(1)	ii)	f(-2)	iii)	f(a)
iv)	$f(b^2)$	v)	f(a - b)	vi)	$f(x^3 - x)$

b) $g: \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $x \mapsto g(x) = \frac{x^2}{x+1}$

Determine the following values:

i)	g(2)	ii)	g(-3)	iii)	g(a)
iv)	$g(b^2)$	v)	g(a - b)	vi)	$g\left(\frac{x^2}{x+1}\right)$

3.4 The graph of a function f ist given as follows:



a) State the value of f(-1).

b) Estimate the value of f(2).

c) For what values of x is f(x) = 2?

d) Estimate the values of x such that f(x) = 0.

e) State the domain D of f.

f) State the range E of f.

3.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The range of the function f: $\{x: x \in \mathbb{R} \text{ and } x \geq 4\} \rightarrow \mathbb{R}$, $x \mapsto y = f(x) = \sqrt{x - 4}$, is the set ...

- ☐ ... $\{x: x \in \mathbb{R} \text{ and } x \geq 4\}$
- ☐ ... $\{y: y \in \mathbb{R} \text{ and } y \geq 4\}$
- ☐ ... $\mathbb{R}$
- ☐ ... $\mathbb{R}_0^+$

b) f cannot be a function if ...

- ☐ ... the domain of f is no number set.
- ☐ ... the codomain of f contains more elements than the domain of f.
- ☐ ... the domain of f contains more elements than the codomain of f.
- ☐ ... at least one element of the domain of f has more than one image.

c) If f is such that $f(x) = x$ for all elements x of the domain, it can be concluded that ...

- ☐ ... the domain of f is the same set as the codomain of f.
- ☐ ... the range of f is the same set as the codomain of f.
- ☐ ... the domain and the codomain of f contain the same number of elements.
- ☐ ... the domain and the range of f contain the same number of elements.

Answers

3.1 a) no function
No element (instead of exactly one element) of B is associated to one of the elements of D.

b) function

c) no function
Two elements (instead of exactly one element) of B are associated to one of the elements of D.

d) function

e) no function
More than one element (instead of exactly one element) of B are associated to some elements of D.

f) no function
Many elements (instead of exactly one element) of B are associated to each element of D.

g) function

h) function

i) no function
Two elements (instead of exactly one element) of $\mathbb{R}$ are associated to each element of $\mathbb{R}^+$.

j) function

3.2 a) $E = \{A, D, F, J, M, N, O, S\}$

b) $E = \{Berlin, Vienna, Vaduz, Rome, Paris\}$

c) $E = B$

d) $E = \mathbb{R}_0^+$

Notice:
- $\mathbb{R}_0^+$ is the set of all positive real numbers, including zero, i.e. $\mathbb{R}_0^+ = \{x: x \in \mathbb{R} \text{ and } x \geq 0\}$.

3.3 a) i) $f(1) = 1^3 - 1 = 0$

ii) $f(-2) = (-2)^3 - (-2) = -6$

iii) $f(a) = a^3 - a$

iv) $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$

v) $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$

vi) $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$

b) i) $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$

ii) $g(-3) = \frac{(-3)^2}{-3+1} = -\frac{9}{2}$

iii) $g(a) = \frac{a^2}{a+1}$

iv) $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$

v) $g(a - b) = \frac{(a-b)^2}{(a-b)+1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$

vi) $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right)+1} = \frac{x^4}{x^3 + 2x^2 + 2x + 1}$

3.4 a) $f(-1) = -2$

b) $f(2) \approx 2.8$

c) $x_1 = -3$, $x_2 = 1$

d) $x_1 \approx -2.5$, $x_2 \approx 0.3$

e) $D = \{x: x \in \mathbb{R} \text{ and } -3 \leq x \leq 3\} = [-3,3]$

f) $E = \{y: y \in \mathbb{R} \text{ and } -2 \leq y \leq 3\} = [-2,3]$

3.5 a) 4th statement

b) 4th statement

c) 4th statement