Exercises 6 Linear function and equations
Linear systems of equations

Objectives

- be able to solve a linear system of equations.
- be able to treat applied tasks by means of linear systems of equations.

Problems

6.1 Solve the following systems of equations:

a) $4x + 3y = 14$
$2x - y = 12$

b) $- 4a - b = 40$
$a + 5b = 9$

c) $12x + 9y = 15$
$4x + 3y = 5$

d) $a - 4b = 3$
$- 5a + 20b = 10$

e) $2p - 6q = 6$
$5p + 3q = 42$

f) $2x + 3y + 5 = 5x + 6y - 1$
$x - 4y - 2 = 2x - 2y$

g) $3(x + 5) = 2(2y - 1)$
$4(3x - 6) = 3(y + 4)$

h) $(x + 5)(y + 1) = (x + 8)(y - 3)$
$(x - 3)(y - 1) = (x - 1)(y + 3)$

i) $2(2a + 3b) = 3(3a - b) + 5$
$4(3a - 4b) = 2(a + b) - 10$

6.2 Find the equation of the linear function whose graph contains the two points P and Q:

a) P(5|-3) Q(-2|1)

b) P(2|-3) Q(-1|-4)

c) P(3|-7) Q(3|-9)

6.3 Find the intersection point of the graphs of the two linear functions f and g:

a) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = -3x + \frac{5}{4}$ $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = g(x) = -x - 1$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = 2x + \frac{5}{4}$ $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = g(x) = 2x - 1$

6.4 Find out whether the graphs of the linear functions f, g, and h have a point P in common.

a) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = x + 1$ $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = g(x) = -\frac{x}{2} - 2$ $h: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = h(x) = \frac{5}{3}x + \frac{7}{3}$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = \frac{1}{6}x + \frac{3}{2}$ $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = g(x) = -\frac{2}{3}x + 2$ $h: \mathbb{R} \to \mathbb{R}$, $x \mapsto y = h(x) = 2x - 3$

6.5 Hotelier A says to hotelier B: "If three quarters of your hotel guests moved to my hotel, I would host 100 guests." Hotelier B replies: "If half of your guests moved to my hotel, I would host 100 guests."

How many guests do A and B host in their hotels?

6.6 The (non-linear) equation $ax^2 + bx = 1$ has the solution set $S = \{2, 3\}$, i.e. the equation has the two solutions $x_1 = 2$ and $x_2 = 3$.

Determine the values of the parameters a and b.

6.7 3000 CHF are awarded to three winners. The first prize is 5/3 of the second one, whereas the second prize is 3/2 of the third one.

Determine the values of the three prizes.

6.8 In a family, the mother is 32 years older than her daughter, whereas the father is 26 years older than his son. In a sum, the mother and her daughter are 10 years older than the father. The difference of the son's and the daughter's ages is twice as much as the difference of the two parents' ages.

Determine the age of each family member.

6.9 Red Tide and Blue Flake are planning new lines of skis.

Red Tide
For the first year, the fixed costs for setting up production are 90'000 CHF. The variable costs for producing each pair of skis are estimated at 160 CHF, and the selling price will be 510 CHF per pair. It is projected that 3000 pairs will sell the first year.

Blue Flake
For the first year, the fixed costs for setting up production are 80'000 CHF. The variable costs for producing each pair of skis are estimated at 160 CHF, and the selling price will be 500 CHF per pair. It is projected that 3500 pairs will sell the first year.

How many pairs of skis must both Red Tide and Blue Flake sell in order to realise the same profit?
What is the profit?

6.10 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) A solution of a linear system of equations ...

- ☐ ... is not necessarily a solution of each equation of the system.
- ☐ ... is not necessarily an element of the solution set.
- ☐ ... consists of two solutions if there are two equations in the system.
- ☐ ... consists of a pair of two real numbers if there are two equations in the system.

b) A solution of a linear system of equations ...

- ☐ ... always corresponds to a common point of graphs of linear functions.
- ☐ ... always corresponds to the intersection point of exactly two straight lines.
- ☐ ... always corresponds to a point in the three-dimensional room.
- ☐ ... is the only solution if the graphs of the corresponding linear functions are parallel.

c) If a system of linear equations has the solution $(x,y) = (2,3)$, it can be concluded that the system ...

- ☐ ... contains more variables than equations.
- ☐ ... contains two equations.
- ☐ ... contains two variables.
- ☐ ... has two solutions.

Answers

6.1 a) $(x, y) = (5, -2)$

b) $(a, b) = (-11, 4)$

c) infinitely many solutions
$(x, y) = \left(x, \frac{5 - 4x}{3}\right)$ $(x \in \mathbb{R})$
$S = \left\{\left(x, \frac{5 - 4x}{3}\right) : x \in R\right\}$

d) no solution
$S = \{ \}$

e) $(p, q) = (15/2, 3/2)$

f) $(x, y) = (6, -4)$

g) $(x, y) = (5, 8)$

h) $(x, y) = (-2, 7)$

i) infinitely many solutions
$(a, b) = \left(a, \frac{5(1 + a)}{9}\right)$ $(a \in \mathbb{R})$
$S = \left\{\left(a, \frac{5(1 + a)}{9}\right) : a \in R\right\}$

6.2 a) $y = f(x) = -\frac{4}{7}x - \frac{1}{7}$

Hints:
- The equation of a linear function is $y = f(x) = ax + b$
- $P(5|-3)$ and $Q(-2|1)$ are points of the graph of the linear function. Therefore, the coordinates of P and Q must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$ and $1 = f(-2) = a \cdot (-2) + b$
- Therfore, solve the following system of equations:

$$-3 = 5a + b$$
$$1 = -2a + b$$

b) $y = f(x) = \frac{1}{3}x - \frac{11}{3}$

c) slope is not defined, therefore no function

6.3 a) $P(9/8 \mid -17/8)$

Hints:
- Let $P(x_1|y_1)$ be an intersection point of the graphs of the two functions f and g. Then, the coordinates x_1 and y_1 must fulfil the two equations $y_1 = f(x_1)$ and $y_1 = g(x_1)$.
- Therefore, solve the following system of equations:

$$y = -3x + \frac{5}{4}$$
$$y = -x - 1$$

b) no intersection point as graphs are parallel

6.4 a) $P(-2|-1)$

b) no intersection point P in common
graphs of f and g intersect at $P(3/5 \mid 8/5)$, however graph of h does not contain P

6.5 (see next page)

6.5 A: 40 guests B: 80 guests

Hints:
- Convert the two statements of the hoteliers into two equations, i.e. into a system of two equations, where the numbers of guests in the two hotels are the variables.
- Solve the system of equations.

6.6 $a = -\frac{1}{6}$ $b = \frac{5}{6}$

6.7 1^{st} prize = 1500 CHF 2^{nd} prize = 900 CHF 3^{rd} prize = 600 CHF

6.8 (f, m, s, d) := (father's age, mother's age, son's age, daughter's age)

2 solutions: $(f, m, s, d)_1 = (54, 48, 28, 16)$
$(f, m, s, d)_2 = (38, 40, 12, 8)$

6.9 Red Tide
Total costs $C_1(x) = 160x + 90'000$
Revenue $R_1(x) = 510x$
Profit $P_1(x) = R_1(x) - C_1(x) = 350x - 90'000$

Blue Flake
Total costs $C_2(x) = 160x + 80'000$
Revenue $R_2(x) = 500x$
Profit $P_2(x) = R_2(x) - C_2(x) = 340x - 80'000$

$P_2(x) = P_1(x)$

$\Rightarrow$ $x = 1000$, $P_1(1000) = P_2(1000) = 260'000$
1000 pairs of skis, profit = 260'000 CHF

6.10 a) 4^{th} statement

b) 1^{st} statement

c) 3^{rd} statement