

# Indefinite integral

Ex.: Financial mathematics

Given the marginal cost function  $C'$  for the production of a commodity:

$$C'(x) = 3x + 50$$

What is the cost function  $C$ ?

$$C(x) = \dots ?$$

## General problem

Given a function  $f$ . What function  $F$  is such that  $F' = f$ ?

Ex.:  $f(x) = 2x$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = x^2 && \text{as } F_1'(x) = 2x = f(x) \\ & F_2(x) = x^2 + 1 && \text{as } F_2'(x) = 2x + 0 = 2x = f(x) \\ & F_3(x) = x^2 - 4 && \text{as } F_3'(x) = 2x + 0 = 2x = f(x) \\ & \dots && \\ & F(x) = x^2 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 2x + 0 = 2x = f(x) \end{aligned}$$

$$f(x) = 8x^3$$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = 2x^4 && \text{as } F_1'(x) = 8x^3 = f(x) \\ & F_2(x) = 2x^4 + 5 && \text{as } F_2'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & F_3(x) = 2x^4 - 11 && \text{as } F_3'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & \dots && \\ & F(x) = 2x^4 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 8x^3 + 0 = 8x^3 = f(x) \end{aligned}$$

## Definitions

$F$  is called an **antiderivative** of  $f$  if its derivative  $F'$  is equal to  $f$ , i.e.  $F'(x) = f(x)$ .

The set of all antiderivatives of the function  $f$  is called the **indefinite integral** of  $f$ , denoted  $\int f(x) \, dx$ .

Ex.:  $f(x) = 8x^3$

The functions  $F_1, F_2, F_3, \dots$  with  $F_1(x) = 2x^4, F_2(x) = 2x^4 + 5, F_3(x) = 2x^4 - 11, \dots$  are all antiderivatives of  $f$ .

We therefore write  $\int f(x) \, dx = \int 8x^3 \, dx = 2x^4 + C$

$$f(x) = 12x^2$$

$$\int f(x) \, dx = \int 12x^2 \, dx = 4x^3 + C$$

$$\int 2x \, dx = x^2 + C$$

$$\int 3 e^{3x} \, dx = e^{3x} + C$$

$C (C \in \mathbb{R})$  is called the **integration constant**.