

Formulary

1. Algebra

Binomial formulae

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Powers and roots

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{n \cdot m} = a^{m \cdot n} = (a^m)^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \sqrt[n]{a^m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

2. Logarithms

$$a^x = y \Leftrightarrow x = \log_a(y)$$

$$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$$

$$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$

$$\log_a(u^v) = v \cdot \log_a(u)$$

3. Functions and equations

Linear function

$$y = f(x) = ax + b$$

$$\text{slope } a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic function (graph = parabola)

$$y = f(x) = ax^2 + bx + c$$

general form

$$y = f(x) = a(x - u)^2 + v$$

vertex form, vertex $V(u|v)$

Quadratic equation

$$ax^2 + bx + c = 0$$

general form

$$\Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exponential function

$$y = f(x) = c \cdot a^x$$

a = growth factor

4. Financial mathematics

r	interest rate per compounding period
n	number of compounding periods
q	interest/growth factor, $q = 1 + r$
C_0	initial capital
C_n	capital after n compounding periods
Ordinary annuity	payments are made at the end of the compounding period
Annuity due	payments are made at the beginning of the compounding period
p	payment before/after each compounding period (all payments are equal)
A_n	value of an annuity after n compounding periods (starting with $C_0 = 0$)
A_0	initial value of an annuity (ending with $C_n = 0$)

Interest

Simple interest

$$C_n = C_0 (1 + nr) \quad \Rightarrow \quad C_0 = \frac{C_n}{1 + nr} \quad r = \frac{\frac{C_n}{C_0} - 1}{n} \quad n = \frac{\frac{C_n}{C_0} - 1}{r}$$

Compound interest

$$C_n = C_0 q^n \quad \Rightarrow \quad C_0 = \frac{C_n}{q^n} \quad q = \sqrt[n]{\frac{C_n}{C_0}} \quad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(q)}$$

$$C_n = C_0 (1 + r)^n \quad \Rightarrow \quad C_0 = \frac{C_n}{(1 + r)^n} \quad r = \sqrt[n]{\frac{C_n}{C_0}} - 1 \quad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(1 + r)}$$

r_a	nominal annual interest rate
r_a^*	effective annual interest rate
m	number of compounding periods per year

$$\Rightarrow \quad r = \frac{r_a}{m}$$

$$r_a^* = \left(1 + \frac{r_a}{m}\right)^m - 1$$

$$C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n \quad \Rightarrow \quad C_0 = \frac{C_n}{\left(1 + \frac{r_a}{m}\right)^n}$$

$$r_a = m \left(\sqrt[n]{\frac{C_n}{C_0}} - 1 \right)$$

$$n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a\left(1 + \frac{r_a}{m}\right)}$$

Annuity

Ordinary annuity

$$A_n = p \frac{q^n - 1}{q - 1} \quad \Rightarrow \quad p = \frac{A_n(q - 1)}{q^n - 1} \quad n = \frac{\log_a \left(\frac{A_n(q - 1)}{p} + 1 \right)}{\log_a(q)}$$

$$A_0 = p \frac{q^n - 1}{q^n(q - 1)} \quad \Rightarrow \quad p = \frac{A_0 q^n(q - 1)}{q^n - 1} \quad n = \frac{\log_a \left(\frac{p}{p - A_0(q - 1)} \right)}{\log_a(q)}$$

Annuity due

$$A_n = pq \frac{q^n - 1}{q - 1} \quad \Rightarrow \quad p = \frac{A_n(q - 1)}{q(q^n - 1)} \quad n = \frac{\log_a \left(\frac{A_n(q - 1)}{pq} + 1 \right)}{\log_a(q)}$$

$$A_0 = p \frac{q^n - 1}{q^{n-1}(q - 1)} \quad \Rightarrow \quad p = \frac{A_0 q^{n-1}(q - 1)}{q^n - 1} \quad n = \frac{\log_a \left(\frac{pq}{pq - A_0(q - 1)} \right)}{\log_a(q)}$$

5. Differential calculus

Derivative	$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$		
Constant function	$f(x) = c \quad (c \in \mathbb{R})$	$\Rightarrow$	$f'(x) = 0$
Power function	$f(x) = x^n \quad (n \in \mathbb{R})$	$\Rightarrow$	$f'(x) = n \cdot x^{n-1}$
Exponential function	$f(x) = a^x \quad (a \in \mathbb{R}^+ \setminus \{1\})$	$\Rightarrow$	$f'(x) = a^x \cdot \ln(a)$
	$f(x) = e^x$	$\Rightarrow$	$f'(x) = e^x$
	$f(x) = e^{f_1(x)}$	$\Rightarrow$	$f'(x) = f_1'(x) \cdot e^{f_1(x)}$
Coefficient rule	$f(x) = c \cdot f_1(x) \quad (c \in \mathbb{R})$	$\Rightarrow$	$f'(x) = c \cdot f_1'(x)$
Sum rule	$f(x) = f_1(x) \pm f_2(x)$	$\Rightarrow$	$f'(x) = f_1'(x) \pm f_2'(x)$
Product rule	$f(x) = f_1(x) \cdot f_2(x)$	$\Rightarrow$	$f'(x) = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$

6. Integral calculus

Constant function	$\int c \, dx = cx + C$	$(c \in \mathbb{R}, C \in \mathbb{R})$
Power function	$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$	$(n \in \mathbb{R} \setminus \{-1\}, C \in \mathbb{R})$
	$\int x^{-1} \, dx = \ln(x) + C$	$(C \in \mathbb{R})$
Exponential function	$\int e^x \, dx = e^x + C$	$(C \in \mathbb{R})$
	$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$	$(k \in \mathbb{R} \setminus \{0\}, C \in \mathbb{R})$
Coefficient rule	$\int c \cdot f(x) \, dx = c \cdot \int f(x) \, dx$	$(c \in \mathbb{R})$
Sum rule	$\int (f_1(x) \pm f_2(x)) \, dx = \int f_1(x) \, dx \pm \int f_2(x) \, dx$	
Definite integral	$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$	